INFORMATION SYSTEMS LABORATORY

DEPARTMENT OF ELECTRICAL ENGINEERING
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OFFICE OF NAVAL RESEARCH
FINAL TECHNICAL REPORT
for
Linearization of Nonlinear Systems

1 December 1985 — 30 November 1988

N00014-86-K-0112

Task No. NR372-176/85

Young Investigator Award

Stephen P. Boyd
Information Systems Laboratory
Department of Electrical Engineering
Stanford University
Stanford, CA 94305

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1 Equipment

As proposed, the ONRYI equipment grant was used in the first year of the award to install a small cluster of 68020-based SUN-3 workstations. These machines were used extensively for the ONRYI research, and also played a very important (and unplanned) role in the computing environment of ISL. These were the first workstations to appear in ISL—as a result, over the next few years, other faculty members and ISL acquired workstations. ISL now has over 35 workstations, and every Ph.D. student has easy access to a workstation. A good knowledge of engineering computation is now universal among ISL graduate students. This new state of affairs is in part due to the ONRYI equipment grant.

2 Work Accomplished

The work accomplished during the ONRYI award spans several areas, all related to computational aspects of systems design or analysis.

2.1 Low Crest-Factor Signals

In [Boy86b], we have made a detailed study of the problem of low crest-factor signals, based on the techniques used in ~~earlier work on~~ harmonic probing of nonlinear systems. Particularly interesting here is the numerical evidence suggesting that the phases used ~~in the original work~~ exceed the performance achievable with the Shapiro-Rudin phases, which can be proved to yield bounded crest factor for multitone signals containing an arbitrarily large number of tones. This work has practical application for frequency response measurements of *linear systems*, and of course is critical for measurements of small distortions in nonlinear systems.

2.2 Bounding and Computing Gains of Systems

The crest factor problem relates two norms, L^2 and L^∞, of a signal; comparing the *gains* of an operator with respect to two norms is much harder. Here a new bound was derived in [BD87].

In [BBK89], a robust numerical method of computing the H_∞ norm of a transfer matrix is developed. The method is applicable to many other system-theoretic quantities and numerical optimization of systems.

2.3 Numerical Determination of Performance Limits

A new computational method for the design of linear controllers was developed in [BBB*88]. In this work, a practical method is proposed for taking advantage of recent results which parameterize all the controllers which stabilize a given plant, or more importantly, all the I/O maps achievable with controllers which stabilize the plant. A *compiler* accepts as input a description of the plant, nominal controller, and design specifications (expressed in a *Control Specification Language*) and as output produces a large convex program which may be solved numerically to yield the desired controller or filter. The compiler **qdes** was implemented.

Absolute performance limits can be numerically determined with **qdes**; [BB89] reports some of these results. A long review article on this new method will appear in the IEEE Proceedings in [BBN89].

In [Boy87], methods of computing optimal open-loop stable controllers are developed.

2.4 Computational Stability Analysis

In [BY89], a new method of computational stability analysis of systems is derived. ~~It is shown that~~ many system stability and robustness problems can be reduced to the question of when there is a quadratic Lyapunov function of a certain structure which establishes stability of $\dot{x} = Ax$ for some appropriate A. The existence of such a Lyapunov function can be determined by solving a nondifferentiable convex program. We present several numerical methods for these optimization problems, and a simple numerical example is given.

In [Boy86b], it is shown how parametric and nonparametric uncertainties in systems can be transformed into each each other, so that methods of analysis (for example, thopse described in [BY89]) of either uncertainty can be applied.

3 Awards and Honors

During the tenure of the ONRYI, there were several other awards and honors.

- NSF Presidential Young Investigator Award.
- D. J. Sakrison Memorial Prize for dissertation research.
- Fannie and John Hertz Foundation Doctoral Thesis Prize.
- Invitation to give talk *Linearization of Nonlinear System by I/O Methods,* ISCAS 1987.
- Invitation to give talk *Overview of ℓ_1-optimal Controller Design,* MTNS 1987.
- Appointment to Board of Governors of IEEE Control Systems Society.
- Appointment to Defense Sciences Study Group.

4 Journal Publications during ONRYI Tenure

The following references include *only* journal publications written under ONRYI support, and technical memos and conference proceedings which have not yet appeared in archival journal form. Selected offprints and preprints are attached.

References

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